Rational function

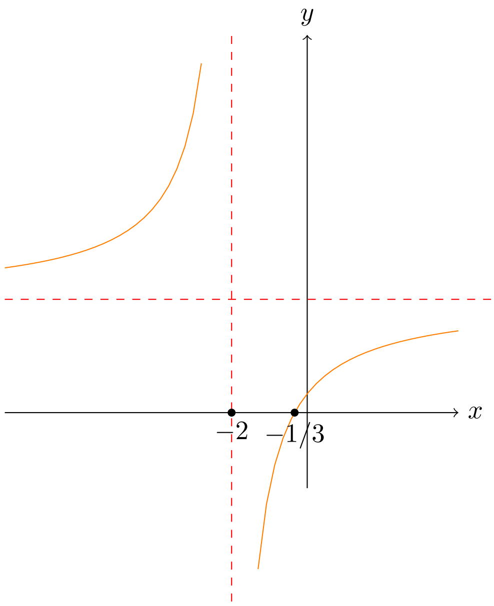
Let the rational function be defined as $f(x) = \dfrac{a}{x-b} + 3$ where $a, b \in \mathbb{R}$

1. Find a and $b \in \mathbb{R}$ so that the graph of $f(x)$ has a vertical asymptote at $x = -2$ and passes through $(-1, -2)$.
2. For the values of a and b found in the previous item, calculate $Dom(f)$, $Im(f)$, and the negativity set of f.

Solution

1. Since we know that the vertical asymptote is at $x = -2$, we know that x can never take that value, as otherwise the function would be divided by 0, so $-2 - b = 0$. Therefore, $b = -2$. To find the value of a we use the given point:
$$-2 = \frac{a}{-1 - (-2)} + 3$$
$$-5 = \frac{a}{1}$$
$$-5 = a$$
Therefore, the function we have is:
$$f(x) = \frac{-5}{x+2} + 3$$

2. The domain consists of the values that x can take. As the only restriction we have in this function is dividing by 0, the domain is all real numbers except -2:
$$Dom : \mathbb{R} - \{-2\}$$
The image consists of all the values that $f(x)$ can take, and these are all real numbers except for the horizontal asymptote (which, by definition, tells us that the function can approach that value but never touch it):
$$Im : \mathbb{R} - \{3\}$$
The asymptote is 3 because the function is composed of two terms, and since $\frac{-5}{x+2}$ can never be 0, the function can never equal 3. Finally, to calculate the negativity set, we remember that these are all the values of x for which the function takes a negative value.
$$\frac{-5}{x+2} + 3 < 0$$
Then, taking into account the graph of the function:



The point where the function intersects the x-axis is obtained by setting the function equal to 0.
$$\frac{-5}{x+2} + 3 = 0$$

$$\frac{-5}{x+2} = -3$$

$$-5 = -3x - 6$$

$$1 = -3x$$

$$-1/3 = x$$

Therefore, the set of negativity is:

$$C_{-} = (-2; -1/3)$$